

Multi-loop Circuits and Kirchoff's laws

It was Qustav Robert Kirchoff who published the first systematic formulation of the principles governing the behavior of electric circuits. Kirchoff's theory includes two laws. The first law deals with the so-called junction point (node). By definition, a point of electric circuit, where three or more conductors are connected together is called the junction point.

1. The first Kirchoff's law states, that the sum of the currents entering or leaving the junction point at any instant of time is equal to zero. In other words, the algebraic sum of the currents at any junction point equals to zero.

$$\sum_i I_i = 0. \quad (4.32)$$

The current, which enters junction point, is considered to be positive. The current, which leaves junction point, is considered to be negative.

For example:

$$I_1 - I_2 - I_3 = 0. \quad (4.33)$$

This law follows from the law of charge conservation. If a charge comes to node must go away.

2. The second Kirchoff's Law deals with closed loops in a branched circuit. According to general form of Ohm's law:

$$\begin{cases} I_1(R_1 + r_1) = \varphi_1 - \varphi_2 + \varepsilon_1 \\ I_2(R_2 + r_2) = \varphi_2 - \varphi_3 + \varepsilon_2 \\ I_3(R_3 + r_3) = \varphi_3 - \varphi_1 + \varepsilon_3 \end{cases} \\ I_1(R_1 + r_1) + I_2(R_2 + r_2) + I_3(R_3 + r_3) = \varepsilon_1 + \varepsilon_2 + \varepsilon_3 \quad (4.34)$$

This law states, that for any closed loop, taken in a branched circuit, the algebraic sum of the products of the currents and the respective resistances is equal to the algebraic sum of all the electromotive forces in the loop. The currents, direction of which coincides with the direction of round of loop, are assumed to be positive. E.m.f. are assumed to be positive if they increase the potential in the direction of round (fig. 4.4). Consequently:

$$\sum_i I_i(R_i + r_i) = \sum_k \varepsilon_k. \quad (4.35)$$

Before talking about what a multi-loop circuit is, it is helpful to define two terms, junction and branch.

A junction is a point where at least three circuit paths meet.

A branch is a path connecting two junctions.

In the circuit below, there are two junctions, labeled *a* and *b*. There are three branches: these are the three paths from *a* to *b*.

In a circuit involving one battery and a number of resistors in series and/or parallel, the resistors can generally be reduced to a single equivalent resistor. With more than one battery, the situation is trickier. If all the batteries are part of one branch they can be combined into a single equivalent battery. Generally, the batteries will be part of different branches, and another method has to be used to analyze the circuit to find the current in each branch. Circuits like this are known as multi-loop circuits.

Finding the current in all branches of a multi-loop circuit (or the e.m.f. of a battery or the value of a resistor) is done by following guidelines known as Kirchoff's rules. These guidelines also apply to very simple circuits.

Kirchoff's first rule: the junction rule. The sum of the currents coming in to a junction is equal to the sum leaving the junction. (Basically this is conservation of charge).

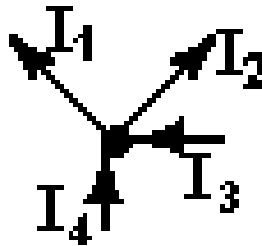


Figure 4.3

Kirchoff's second rule: the loop rule. The sum of all the potential differences around a complete loop is equals zero (energy conservation).

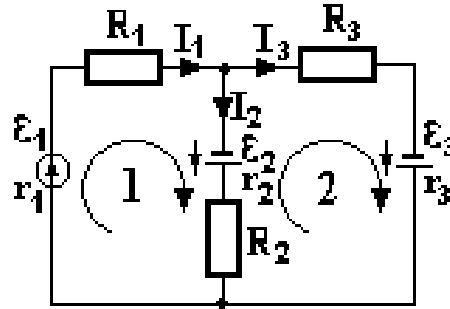


Figure 4.4

There are two different methods for analyzing circuits. The standard method in physics, which is the one followed by the textbook, is the branch current method. There is another method, the loop current method, but we won't worry about that one.

Ohm's Law for Heterogeneous Circuit

Heterogeneous circuit contains the so-called current sources. In order to obtain a current in a conductor a difference of potentials must be maintained on its ends. Devices, enabling to maintain a potential difference are called current sources. Such sources are not of physical nature. They are usually electrochemistry sources. They can increase the potential. Any current source has got two characteristics: the internal resistance r , the electromotive force of a source \mathcal{E} (e.m.f.) Unit of \mathcal{E} is: $[\mathcal{E}] = \text{V}$.

In an external circuit current flows from plus to minus. However in the current source the current flows visa versa: from minus to plus as it is shown on the Fig. You can see that a drop of potential is observed in external circuit. However the increase of potential occurs inside a source. Ohm's law is given by the next formula:

$$I = \frac{\varphi_1 - \varphi_2 + \mathcal{E}}{R + r}, \quad (4.36)$$

$$I(R + r) = \varphi_1 - \varphi_2 + \mathcal{E}. \quad (4.37)$$

The last equation is the general form of Ohm's law.